TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 4 2021

Section 1 Calculator Free

Integration and Applications of Integration

STUDENT'S NAME _____

DATE: Tuesday 27 July **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, Formula Booklet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine the following integrals:

(a) $\int \frac{x^2 - 1}{x} dx$ [2]

(b) $\int \frac{\ln(x^2)}{x} dx$ [3]

2. (9 marks)

Determine the following integrals:

(a) $\int \frac{\sin^2\theta + \cos^2\theta}{\cos 2\theta + \sin^2\theta}\, d\theta$ [3]

(b) $\int \sin^3 x\, dx$ [3]

(c) $\int \frac{x^2}{x-1}\, dx$ [3]

3. (5 marks)

(a) Express $\frac{x+7}{(x+1)(x-2)}$ in the form $\frac{a}{x+1}+\frac{b}{x-2}$. [2]

(b) Hence, determine $\int \frac{x+7}{(x+1)(x-2)}dx$ [3]

4. (6 marks)

Evaluate exactly: $\int_0^{\sqrt{2}} \sqrt{1-\frac{x^2}{4}}\, dx$ using the substitution $x = 2\sin\theta$

TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 4 2021

Section 2 Calculator Assumed

Integration and Applications of Integration

STUDENT'S NAME _____

DATE: Tuesday 27 July **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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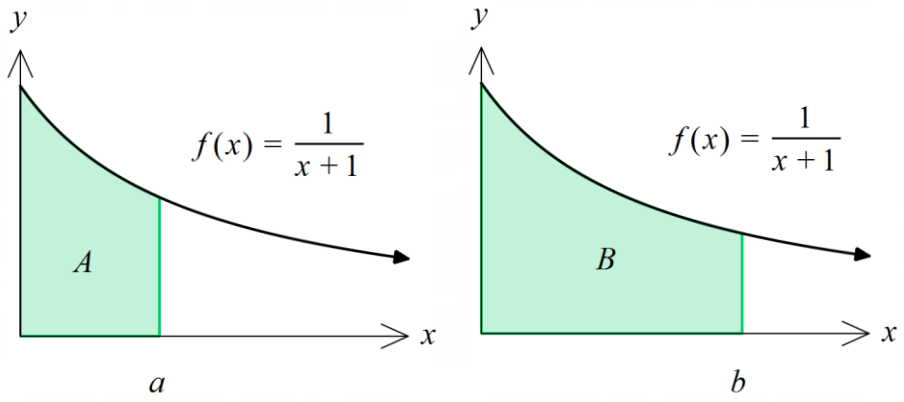
5. (4 marks)

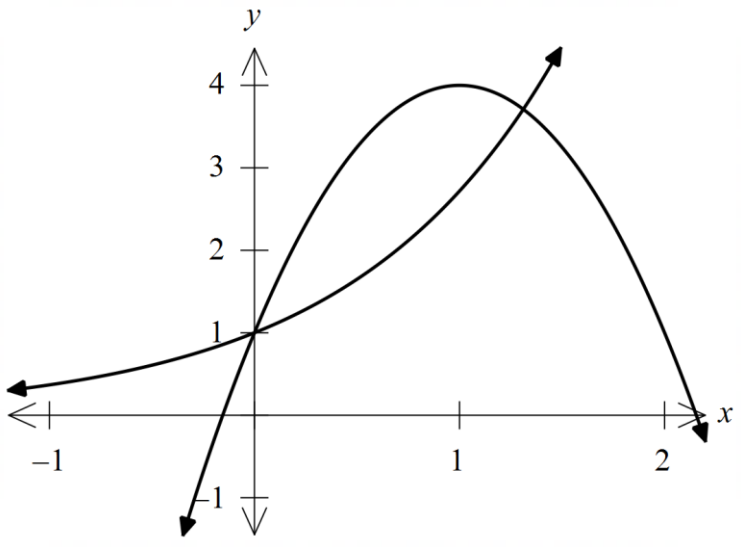
The area labelled B is three times the area labelled A.



Express b in terms of a.

6. (8 marks)

Consider the two functions $f(x) = e^x$ and $g(x) = -3x^2 + 6x + 1$.



(a) (i) Write an integral expression for the approximate enclosed area between the curves. [2]

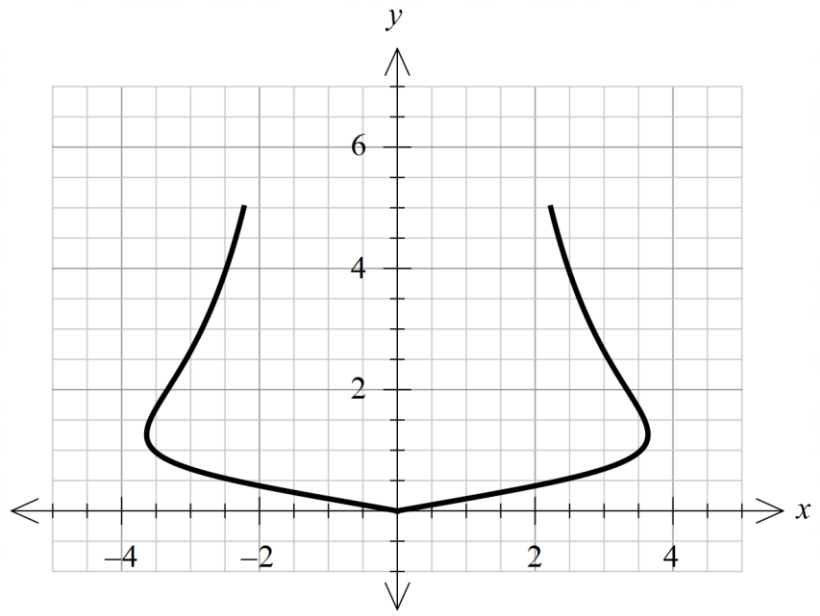
(ii) Calculate the approximate enclosed area. [2]

(b) (i) Write down an integral expression for volume formed when the enclosed region is rotated about the x-axis. [2]

(ii) Calculate the volume formed when the enclosed region is rotated about the x-axis. [2]

7. (5 marks)

The top part of a wine glass is modelled by rotating the graph of $x^2 = y^2(25 - x^2 y)$ from $y = 0$ to $y = 5$ about the y axis as shown below. Dimensions are measured in centimetres.



Calculate, correct to the nearest 0.01 cm, the depth of wine in the glass if it is to contain 75% of its maximum volume.

8. (8 marks)

The table below gives the value of a function obtained from an experiment.

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	-7.6	-10.5

Two different methods are used to approximate $\int_0^6 f(x)\,dx$.

(a) Method 1: Using three equal subintervals, estimate $\int_0^6 f(x)\,dx$ by using trapeziums. [4]

(b) Method 2: The function $g(x) = 0.14x^4 - 1.57x^3 + 4.63x^2 - 4.34x + 9.48$ is used to estimate $f(x)$

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	-7.6	-10.5
$g(x)$	9.48	8.34	9	7.08	1.56	-5.22	-7.56

Calculate $\int_0^6 g(x)\,dx$ [1]

(c) For this question, explain the limitations of each method and comment on which estimate is more accurate. [3]